

PROGRESSION THROUGH CALCULATIONS FOR ADDITION

MENTAL CALCULATIONS

(ongoing)

These are a **selection** of mental calculation strategies from the National Numeracy Strategy Framework:

Mental recall of number bonds

$$6 + 4 = 10$$

$$25 + 75 = 100$$

$$\square + 3 = 10$$

$$19 + \square = 20$$

Use near doubles

$$6 + 7 = \text{double } 6 + 1 = 13$$

Addition using partitioning and recombining

$$34 + 45 = (30 + 40) + (4 + 5) = 79$$

Counting on or back in repeated steps of 1, 10, 100, 1000

$$86 + 57 = 143 \text{ (by counting on in tens and then in ones)}$$

$$460 - 300 = 160 \text{ (by counting back in hundreds)}$$

Add the nearest multiple of 10, 100 and 1000 and adjust

$$24 + 19 = 24 + 20 - 1 = 43$$

$$458 + 71 = 458 + 70 + 1 = 529$$

Use the relationship between addition and subtraction

$$36 + 19 = 55$$

$$19 + 36 = 55$$

$$55 - 19 = 36$$

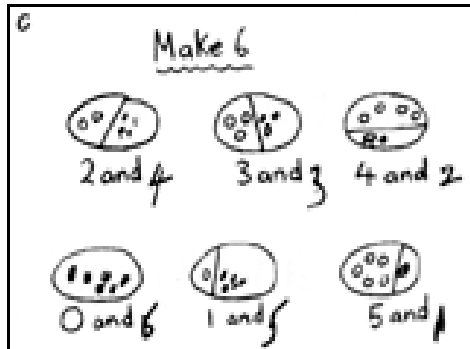
$$55 - 36 = 19$$

MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.

THE FOLLOWING ARE STANDARDS THAT WE EXPECT THE MAJORITY OF CHILDREN TO ACHIEVE.

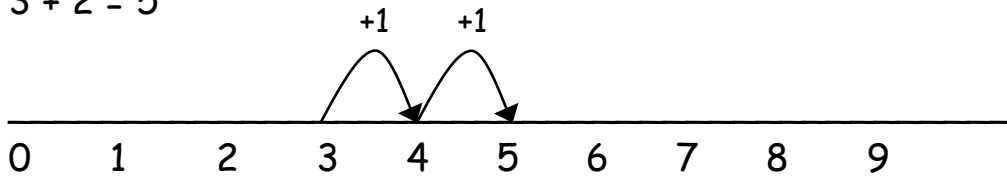
YR and Y1

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.



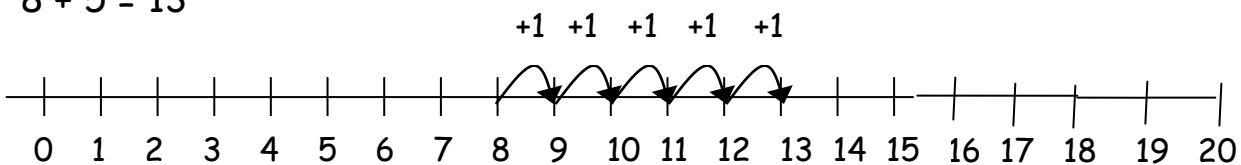
Children use number lines and practical resources to support calculation and teachers *demonstrate* the use of the number line. In Year 1 the number range will extend to numbers from zero to 20.

$$3 + 2 = 5$$

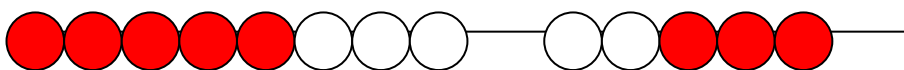


Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones. In Year 1, calculations will extend to include adding one digit numbers to two digit numbers - including from zero.

$$8 + 5 = 13$$



Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.



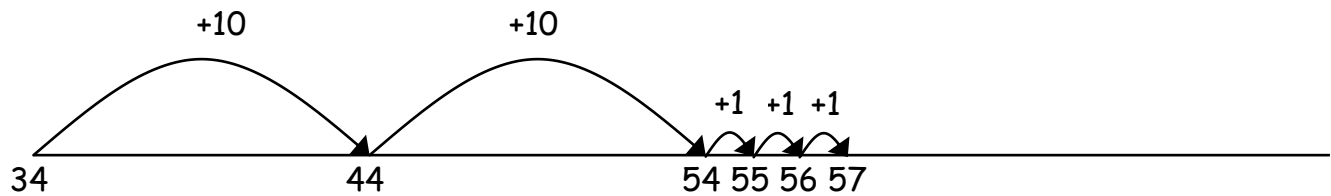
Children will solve missing number problems such as $7 = \square + 2$ using concrete resources.

Y2

The number range in Year 2 will focus on fluency with number bonds to 20 and then extend into related facts to 100. Children will begin to use 'empty number lines' themselves, starting with the larger number and counting on. This will extend to adding 3 one digit numbers.

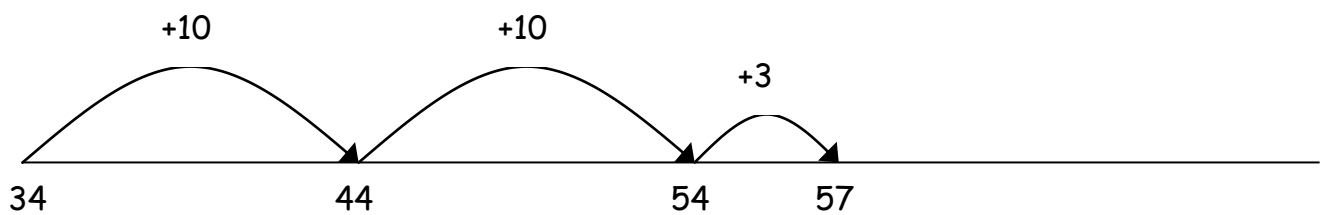
- ✓ First counting on in tens and ones.

$$34 + 23 = 57$$



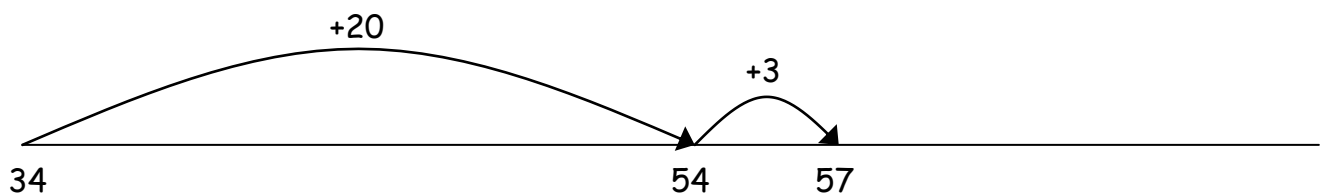
- ✓ Then helping children to become more efficient by adding the ones in one jump (by using the known fact $4 + 3 = 7$).

$$34 + 23 = 57$$



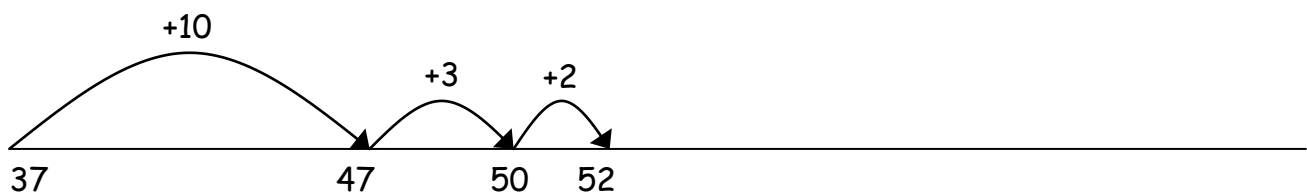
- ✓ Followed by adding the tens in one jump and the ones in one jump.

$$34 + 23 = 57$$

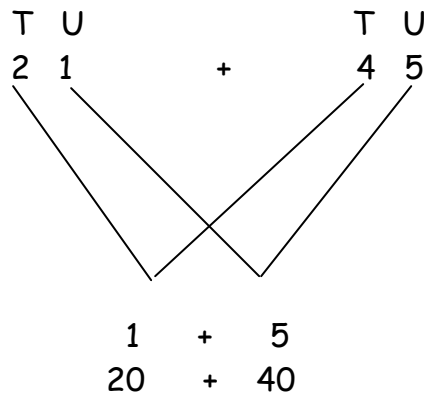


- ✓ Bridging through ten can help children become more efficient.

$$37 + 15 = 52$$



Children will progress to adding 2 two digit numbers by beginning to use written methods which involve partitioning.

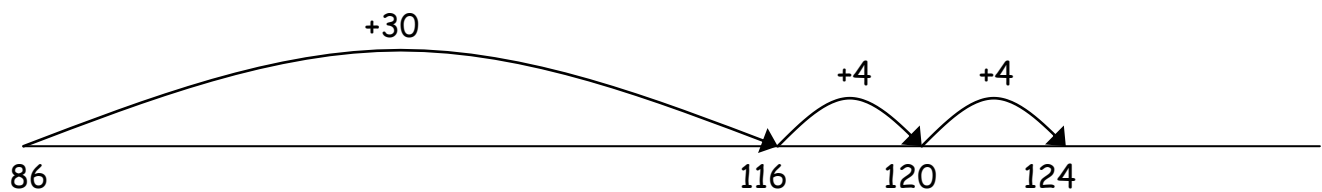


Y3

Children will continue to use empty number lines with increasingly large numbers, including compensation where appropriate.

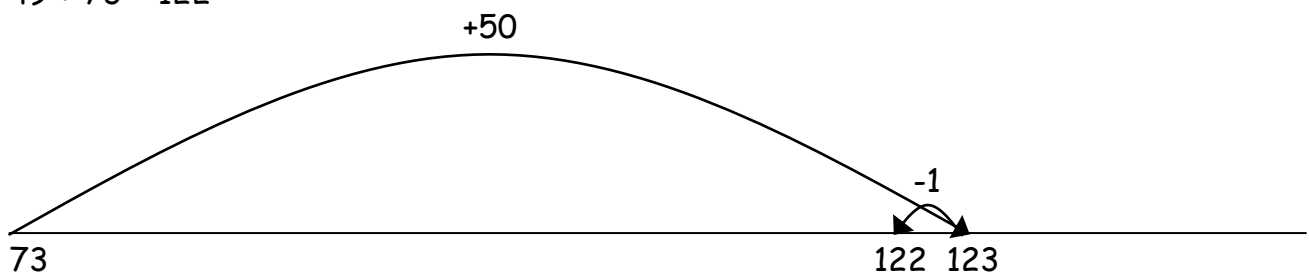
- ✓ Count on from the largest number irrespective of the order of the calculation.

$$38 + 86 = 124$$



- ✓ Compensation

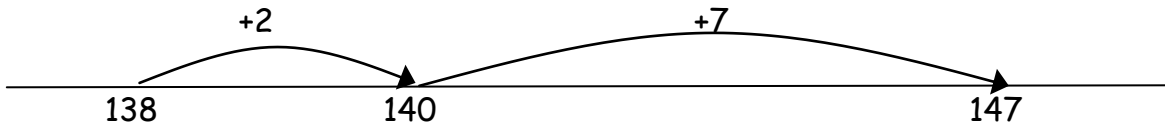
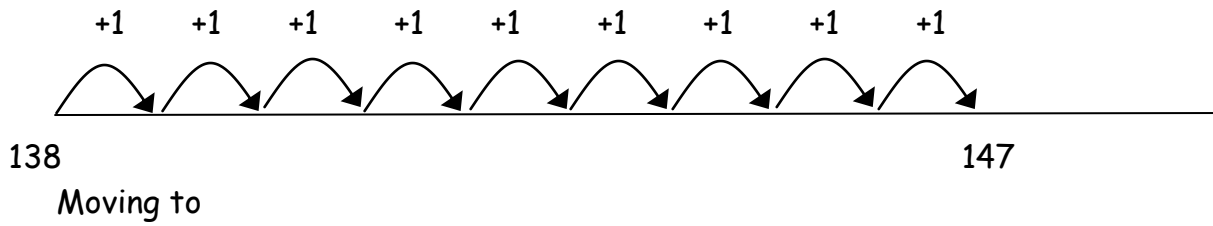
$$49 + 73 = 122$$



Children will mentally add a three digit number and ones, a three digit number and tens and a three digit number and hundreds.

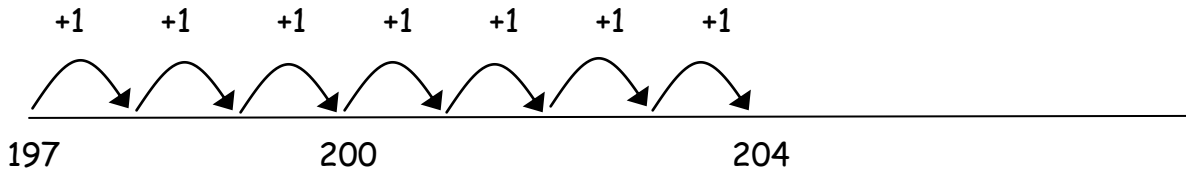
✓ Three digit + ones

$$138+9=147$$



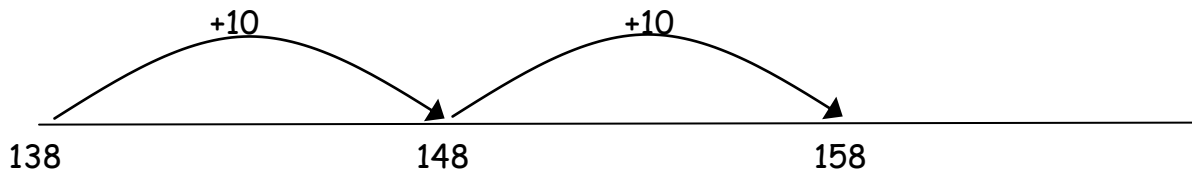
✓ Bridging 100

$$197+7=204$$



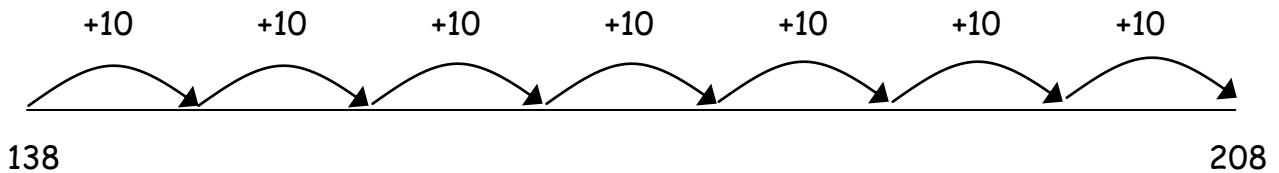
✓ Three digit + tens

$$138+20=158$$

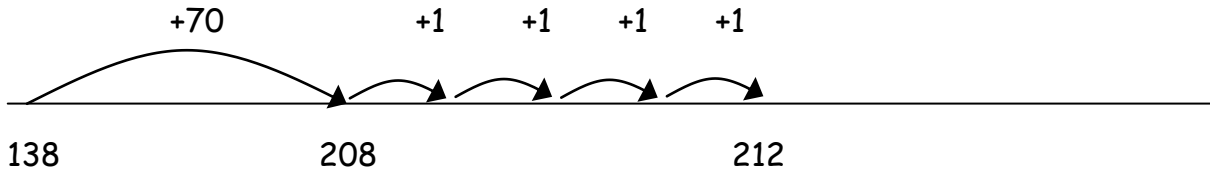


✓ Bridging through one hundreds

$$138+70=208$$

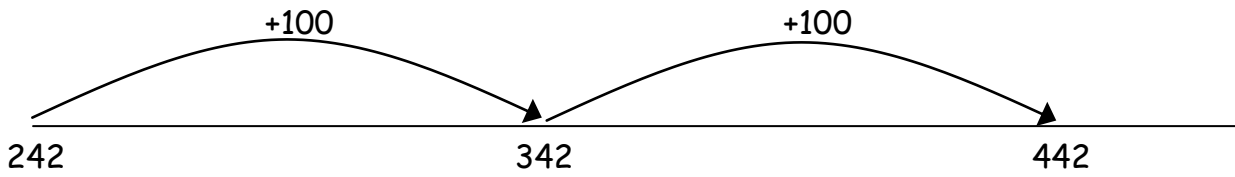


$138+74=212$

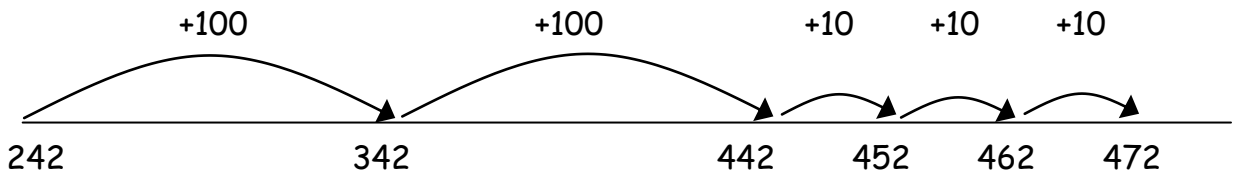


✓ Three digit + hundreds

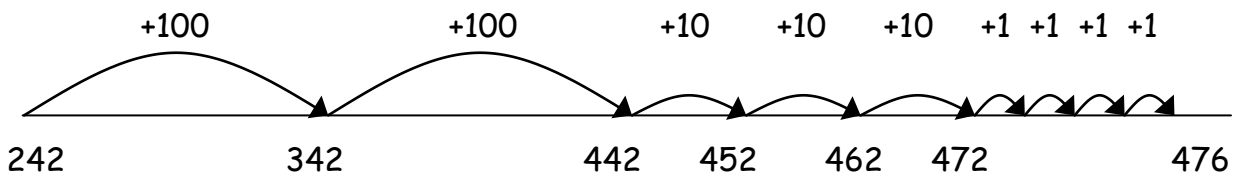
$242+200=442$



$242+230=472$

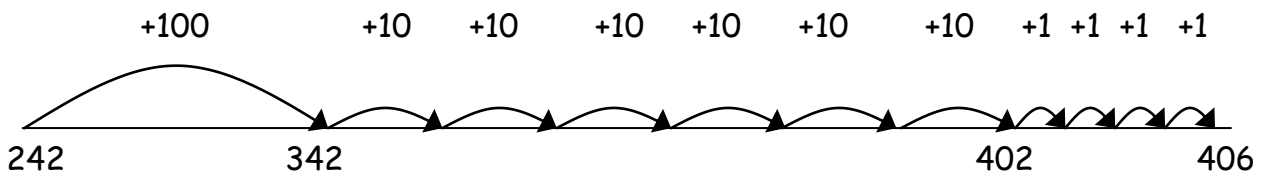


$242+234=476$



✓ Bridging through hundreds

$242+164=406$



Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

Option 1 - Adding most significant digits first, then moving to adding least significant digits.

$$\begin{array}{r} 67 \\ + 24 \\ \hline 80 \text{ (60 + 20)} \\ \underline{11} \text{ (7 + 4)} \\ \hline 91 \end{array}$$

$$\begin{array}{r} 267 \\ + 85 \\ \hline 200 \\ 140 \text{ (60 + 80)} \\ \underline{12} \text{ (7 + 5)} \\ \hline 352 \end{array}$$

Moving to adding the least significant digits first in preparation for 'carrying'.

$$\begin{array}{r} 67 \\ + 24 \\ \hline 11 \text{ (7 + 4)} \\ \underline{80} \text{ (60 + 20)} \\ \hline 91 \end{array}$$

$$\begin{array}{r} 267 \\ + 85 \\ \hline 12 \text{ (7 + 5)} \\ 140 \text{ (60 + 80)} \\ \underline{200} \\ \hline 352 \end{array}$$

Option 2 - Adding the least significant digits first

$$\begin{array}{r} 67 \\ + 24 \\ \hline 11 \text{ (7 + 4)} \\ \underline{80} \text{ (60 + 20)} \\ \hline 91 \end{array}$$

$$\begin{array}{r} 267 \\ + 85 \\ \hline 12 \text{ (7 + 5)} \\ 140 \text{ (60 + 80)} \\ \underline{200} \\ \hline 352 \end{array}$$

From this, children will begin to carry below the line

$$\begin{array}{r} 625 \\ + 48 \\ \hline 673 \\ \hline 1 \end{array}$$

$$\begin{array}{r} 783 \\ + 42 \\ \hline 825 \\ \hline 1 \end{array}$$

$$\begin{array}{r} 367 \\ + 85 \\ \hline 452 \\ \hline 11 \end{array}$$

$$\begin{array}{r} 367 \\ + 185 \\ \hline 552 \\ \hline 11 \end{array}$$

Using similar methods, children will:

- ✓ add several numbers with different numbers of digits;
- ✓ begin to add two or more three-digit sums of money, with or without adjustment from the pence to the pounds;
- ✓ know that the decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. £3.59 + £0.78.

Y4

Children should extend the carrying method to numbers with at least four digits.

$$\begin{array}{r} 587 \\ + 475 \\ \hline 1062 \\ \hline 11 \end{array}$$

$$\begin{array}{r} 3587 \\ + 675 \\ \hline 4262 \\ \hline 111 \end{array}$$

$$\begin{array}{r} 7648 \\ + 1486 \\ \hline 9134 \\ \hline 111 \end{array}$$

Using similar methods, children will:

- ✓ add several numbers with different numbers of digits;
- ✓ begin to add two or more decimal fractions with up to three digits and the same number of decimal places;
- ✓ know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 3.2 m - 280 cm.
- ✓ To be able to convert amounts to the same units in order to calculate, eg 3.2m - 280cm becomes 320cm - 280cm.

Y5 to 6

Children should extend the carrying method to number with any number of digits.

$$\begin{array}{r} 7648 \\ + 1486 \\ \hline 9134 \\ 111 \end{array}$$

$$\begin{array}{r} 6584 \\ + 5848 \\ \hline 12432 \\ 111 \end{array}$$

$$\begin{array}{r} 42 \\ 6432 \\ 786 \\ 3 \\ + 4681 \\ \hline 11944 \\ 121 \end{array}$$

Using similar methods, children will

- ✓ *add several numbers with different numbers of digits;*
- ✓ *begin to add two or more decimal fractions with up to four digits and either one or two decimal places;*
- ✓ *know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. $401.2 + 26.85 + 0.71$.*

+ - + - + - + - + - + - +

By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

- they are not ready.
- they are not confident.

Children should be encouraged to approximate their answers before calculating. Children should be encouraged to check their answers after calculation using an appropriate strategy.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

PROGRESSION THROUGH CALCULATIONS FOR SUBTRACTION

MENTAL CALCULATIONS

(ongoing)

These are a **selection** of mental calculation strategies from the National Numeracy Strategy framework:

Mental recall of addition and subtraction facts

$$10 - 6 = 4$$

$$17 - \square = 11$$

$$20 - 17 = 3$$

$$10 - \square = 2$$

Find a small difference by counting up

$$82 - 79 = 3$$

Counting on or back in repeated steps of 1, 10, 100, 1000

$$86 - 52 = 34 \text{ (by counting back in tens and then in ones)}$$

$$460 - 300 = 160 \text{ (by counting back in hundreds)}$$

Subtract the nearest multiple of 10, 100 and 1000 and adjust

$$24 - 19 = 24 - 20 + 1 = 5$$

$$458 - 71 = 458 - 70 - 1 = 387$$

Use the relationship between addition and subtraction

$$36 + 19 = 55$$

$$19 + 36 = 55$$

$$55 - 19 = 36$$

$$55 - 36 = 19$$

MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.

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YR and Y1

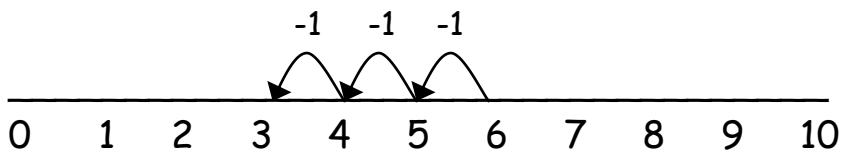
The number range in R and Y1 will be numbers to 20 including 0.

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures etc.

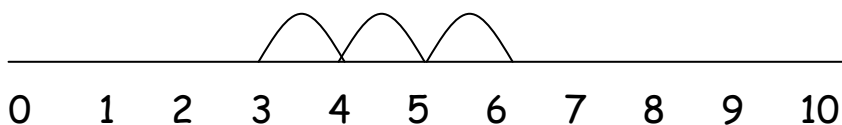


They use number lines and practical resources to support calculation. Teachers *demonstrate* the use of the number line.

$$6 - 3 = 3$$

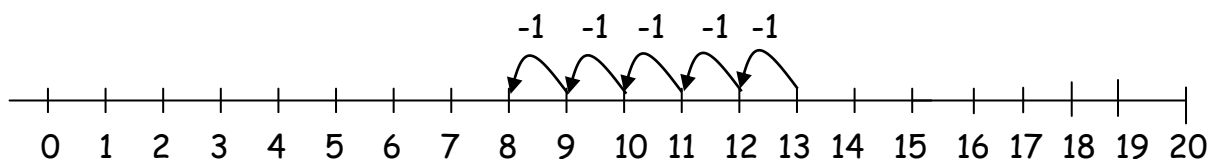


The number line should also be used to show that $6 - 3$ means the 'difference between 6 and 3' or 'the difference between 3 and 6' and how many jumps they are apart.



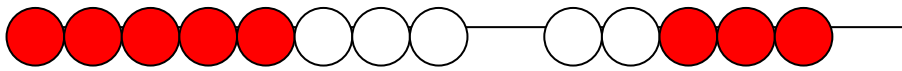
Children then begin to use numbered lines to support their own calculations - using a numbered line to count back in ones.

$$13 - 5 = 8$$



Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.

$$13 - 5 = 8$$



To find the difference, children can also count on using concrete apparatus.

Y2

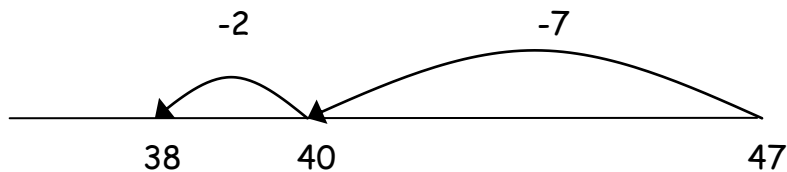
The number range in Y2 will focus on fluency with number bonds to 20 and then extend into related facts to 100.

Children will begin to use empty number lines to support calculations.

Counting back *[NB Counting back can be carried out above or below the numberline].*

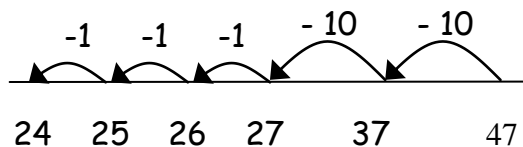
- ✓ Counting back in ones.

$$47 - 9 = 38$$



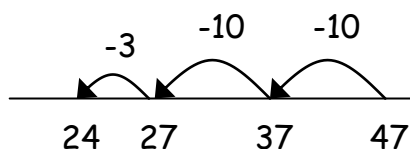
- ✓ Counting back in tens and ones.

$$47 - 23 = 24$$



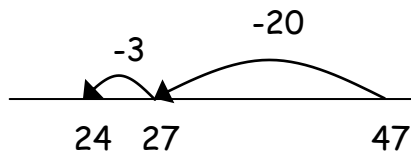
- ✓ Then helping children to become more efficient by subtracting the ones in one jump (by using the known fact $7 - 3 = 4$).

$$47 - 23 = 24$$



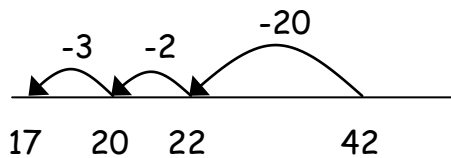
- ✓ Subtracting the tens in one jump and the ones in one jump.

$$47 - 23 = 24$$



- ✓ Bridging through ten can help children become more efficient.

$$42 - 25 = 17$$



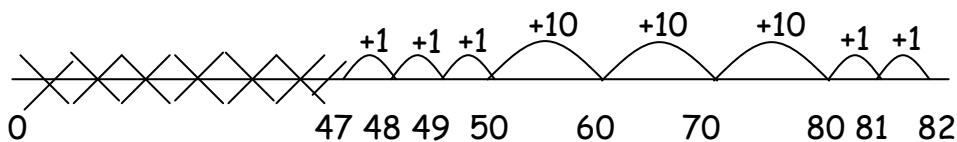
Counting on

If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, it can be more efficient to count on.

Count up from 47 to 82 in jumps of 10 and jumps of 1.

The number line should still show 0 so children can cross out the section from 0 to the smallest number. They then associate this method with 'taking away'.

$$82 - 47 = 35$$



Help children to become more efficient with counting on by:

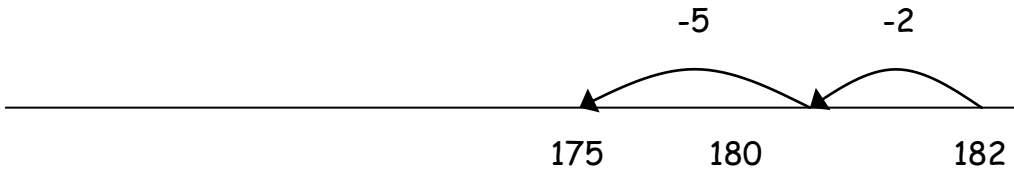
- ✓ Subtracting the ones in one jump;
- ✓ Subtracting the tens in one jump and the ones in one jump;
- ✓ Bridging through ten by partitioning.

Y3

Children will continue to use empty number lines with increasingly large numbers recognising whether it is more efficient to count on or count back.

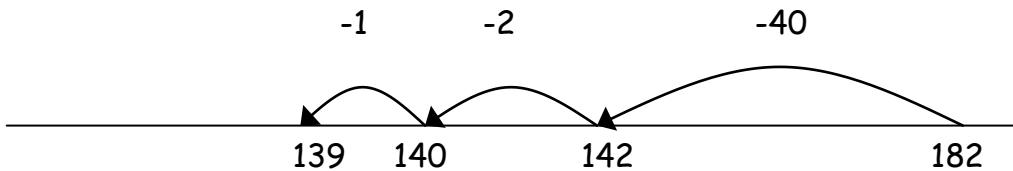
- ✓ A three digit number and ones by counting back.

$$182 - 7 = 175$$

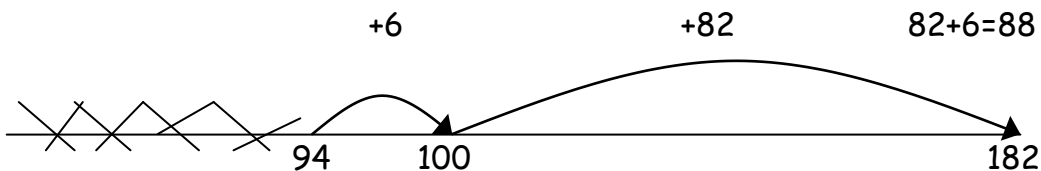


- ✓ A three digit number and tens using counting on or counting back.

$$182 - 43 = 139$$

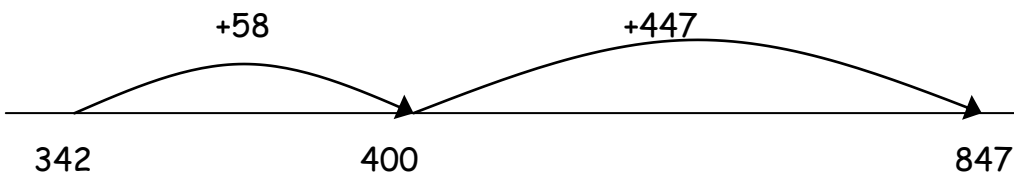


$$182 - 94 = 88$$



- ✓ A three digit number and hundreds using counting on with multiples of 100 as marker.

$$847 - 342 = 505$$



$$\begin{array}{r} 447 + 58 \\ \hline 505 \end{array}$$

400 + 90 + 15 = 505

Or column addition:

$$\begin{array}{r} 447 \\ + 58 \\ \hline 505 \end{array}$$

11

Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

Partitioning and decomposition

This process should be demonstrated using arrow cards to show the partitioning and base 10 materials to show the decomposition of the number.

NOTE When solving the calculation $89 - 57$, children should know that 57 **does NOT EXIST AS AN AMOUNT** it is what you are subtracting from the other number. Therefore, when using base 10 materials, children would need to count out only the 89.

$$\begin{array}{r} 89 \\ - 57 \\ \hline \end{array} = \begin{array}{r} 80 + 9 \\ 50 + 7 \\ \hline 30 + 2 = 32 \end{array}$$

Initially, the children will be taught using examples that do not need the children to exchange.

From this the children will begin to exchange.

$$\begin{array}{r} 71 \\ - 46 \\ \hline \end{array} = \quad =$$

Step 1

$$\begin{array}{r} 70 + 1 \\ - 40 + 6 \\ \hline \end{array}$$

Step 2

$$\begin{array}{r} 60 + 11 \\ - 40 + 6 \\ \hline 20 + 5 = 25 \end{array}$$

The calculation should be read as e.g. take 6 from 1.

This would be recorded by the children as

$$\begin{array}{r} \overset{60}{\cancel{70}} + 11 \\ - 40 + 6 \\ \hline 20 + 5 = 25 \end{array}$$

Children should know that ones line up under ones, tens under tens, and so on.

$$\begin{array}{r} 754 = \\ - 86 \\ \hline \end{array}$$

$$\text{Step 1} \quad \begin{array}{r} 700 + 50 + 4 \\ - \quad \quad 80 + 6 \\ \hline \end{array}$$

$$\text{Step 2} \quad \begin{array}{r} 700 + 40 + 14 \\ - \quad \quad 80 + 6 \\ \hline \end{array} \quad (\text{adjust from } T \text{ to } O)$$

$$\text{Step 3} \quad \begin{array}{r} 600 + 140 + 14 \\ - \quad \quad 80 + 6 \\ \hline 600 + 60 + 8 = 668 \end{array} \quad (\text{adjust from } H \text{ to } T)$$

This would be recorded by the children as

$$\begin{array}{r} \begin{array}{r} 600 \\ \cancel{700} \end{array} + \begin{array}{r} 140 \\ \cancel{50} \end{array} + 14 \\ - \quad \quad 80 + 6 \\ \hline 600 + 60 + 8 = 668 \end{array}$$

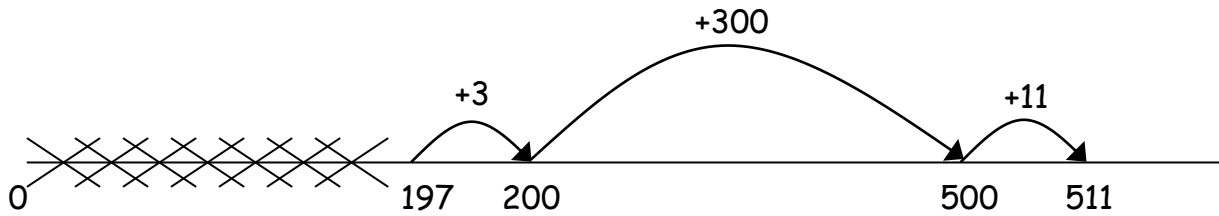
Decomposition

$$\begin{array}{r} 614 \\ 754 \\ - 286 \\ \hline 468 \end{array}$$

Year 4

Where the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, counting on using a number line should be used.

$$511 - 197 = 314$$



Partitioning and decomposition with up to four digits

$$\begin{array}{r} \text{Step 1} \quad 1754 = 1000 + 700 + 50 + 4 \\ \quad \quad \quad - 286 \quad - \quad \quad \quad \underline{200 + 80 + 6} \end{array}$$

$$\begin{array}{r} \text{Step 2} \quad \quad \quad 1000 + 700 + 40 + 14 \quad (\text{adjust from } T \text{ to } O) \\ \quad \quad \quad - \quad \quad \quad \underline{200 + 80 + 6} \end{array}$$

$$\begin{array}{r} \text{Step 3} \quad \quad \quad 1000 + 600 + 140 + 14 \quad (\text{adjust from } H \text{ to } T) \\ \quad \quad \quad - \quad \quad \quad \underline{200 + 80 + 6} \\ 1000 + 400 + 60 + 8 = 1468 \end{array}$$

This would be recorded by the children as

$$\begin{array}{r} 1000 + \overset{600}{\cancel{700}} + \overset{140}{\cancel{50}} + 14 \\ - \quad \quad \quad \underline{200 + 80 + 6} \\ 1000 + 400 + 60 + 8 = 1468 \end{array}$$

Decomposition

$$\begin{array}{r} \begin{array}{r} 6141 \\ \cancel{784} \\ - 86 \\ \hline 668 \end{array} \quad \text{to} \quad \begin{array}{r} 5131 \\ \cancel{6467} \\ - 2684 \\ \hline 3783 \end{array} \end{array}$$

Children should:

- ✓ be able to subtract numbers with up to four digits;
- ✓ begin to find the difference between two decimal fractions with up to three digits and the same number of decimal places;
- ✓ know that decimal points should line up under each other.

- ✓ using this method, children should also begin to find the difference between two three-digit sums of money, with or without 'adjustment' from the pence to the pounds.

For example:

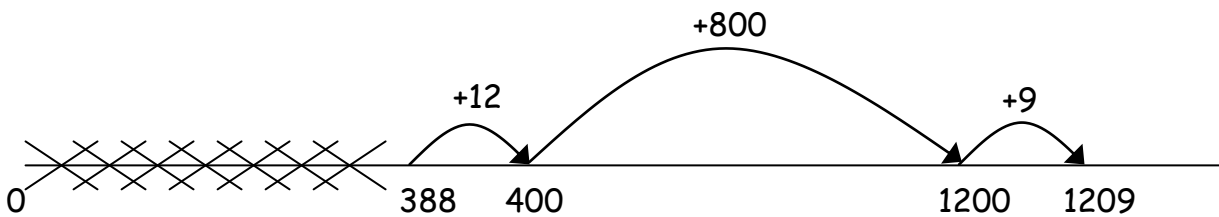
$$\begin{array}{r}
 \text{£}8.95 = 8 + 0.9 + 0.05 \\
 \underline{-\text{£}4.38} \quad - \underline{4 + 0.3 + 0.08} \\
 \\
 = 8 + 0.8 + 0.15 \quad (\text{adjust from } T \text{ to } O) \\
 - 4 + 0.3 + 0.08 \\
 \hline
 4 + 0.5 + 0.07
 \end{array}
 \qquad
 \begin{array}{r}
 \text{leading to} \\
 \\
 \begin{array}{r}
 8.85 \\
 - 4.38 \\
 \hline
 = \text{£}4.57
 \end{array}
 \end{array}$$

Alternatively, children can set the amounts to whole numbers, i.e. 895 - 438 and convert to pounds after the calculation.

NB If your children have reached the concise stage they will then continue this method through into years 5 and 6. They will not go back to using the expanded methods

Where the numbers are involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.

$$1209 - 388 = 821$$



Y5

The number range will extend to more than four digits.

Decomposition

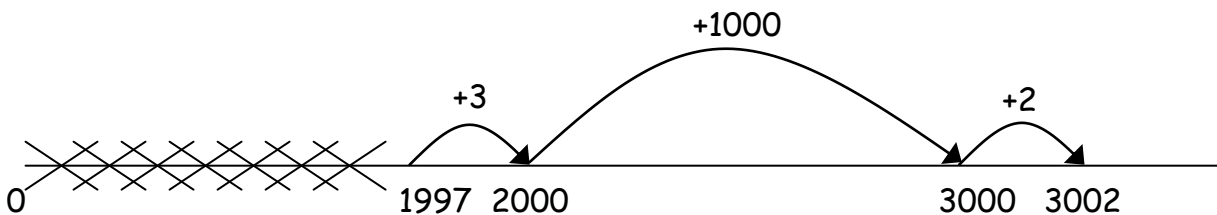
$$\begin{array}{r} ^5 ^{13} ^1 \\ 6467 \\ - 2684 \\ \hline 3783 \end{array}$$

Children should:

- ✓ be able to subtract numbers with different numbers of digits, increasing for example to five digits;
- ✓ be able to subtract two or more decimal fractions with up to three digits and either one or two decimal places;
- ✓ know that decimal points should line up under each other.

Where the numbers are involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.

$$3002 - 1997 = 1005$$



By the end of year 5, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Y6

Children should not be made to go onto the next stage if:

- they are not ready.
- they are not confident.

Children should be encouraged to approximate their answers before calculating. Children should be encouraged to check their answers after calculation using an appropriate strategy.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

PROGRESSION THROUGH CALCULATIONS FOR MULTIPLICATION

MENTAL CALCULATIONS

(ongoing)

These are a **selection** of mental calculation strategies from The National Numeracy Strategy framework:

Doubling and halving

Applying the knowledge of doubles and halves to known facts.

e.g. 8×4 is double 4×4

Using multiplication facts

In Y1, children will count in 2s, 5s and 10s. Tables should be taught everyday from Y2 onwards, either as part of the mental oral starter or other times as appropriate within the day.

Year 2 2 times table
 3 times table
 4 times table
 5 times table
 10 times table

Year 3 2 times table
 3 times table
 4 times table
 5 times table
 6 times table
 8 times table
 9 times table
 10 times table

Years 4, 5 & 6 Derive and recall all multiplication facts up to 12×12

Using and applying division facts

Children should be able to utilise their tables knowledge to derive other facts.

e.g. If I know $3 \times 7 = 21$, what else do I know?

$30 \times 7 = 210$, $300 \times 7 = 2100$, $3000 \times 7 = 21\ 000$, $0.3 \times 7 = 2.1$ etc

Use closely related facts already known

$13 \times 11 = (13 \times 10) + (13 \times 1)$
 $= 130 + 13$
 $= 143$

Multiplying by 10 or 100

Knowing that the effect of multiplying by 10 is a shift in the digits one place to the left.

Knowing that the effect of multiplying by 100 is a shift in the digits two places to the left.

Partitioning

$$23 \times 4 = (20 \times 4) + (3 \times 4)$$

$$= 80 + 12$$

$$= 102$$

Use of factors

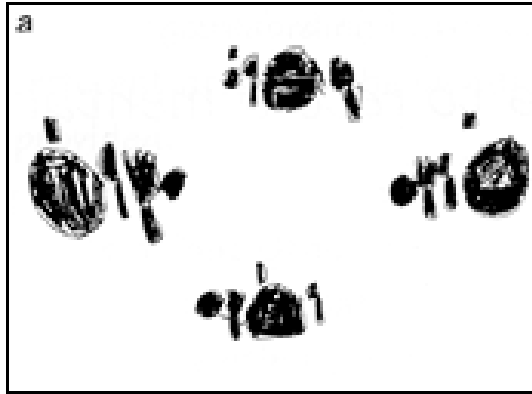
$$8 \times 12 = 8 \times 4 \times 3$$

MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.

THE FOLLOWING ARE STANDARDS THAT WE EXPECT THE MAJORITY OF CHILDREN TO ACHIEVE.

YR and Y1

Children will experience equal groups of objects and will count in 2s, 5s and 10s and begin to count back in 2s, 5s and 10s. They will work on practical problem solving activities involving equal sets/groups/lots.



Y2

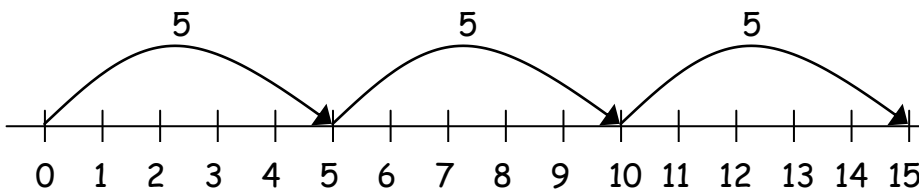
Children will develop their understanding of multiplication and use jottings to support calculation:

✓ **Repeated addition**

3 times 5 is $5 + 5 + 5 = 15$ or 3 lots of 5 or 5×3

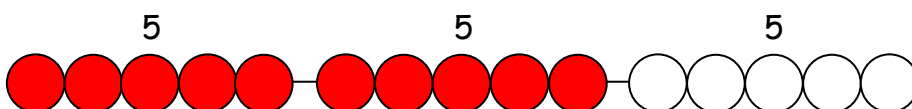
Repeated addition can be shown easily on a number line:

$$5 \times 3 = 5 + 5 + 5$$



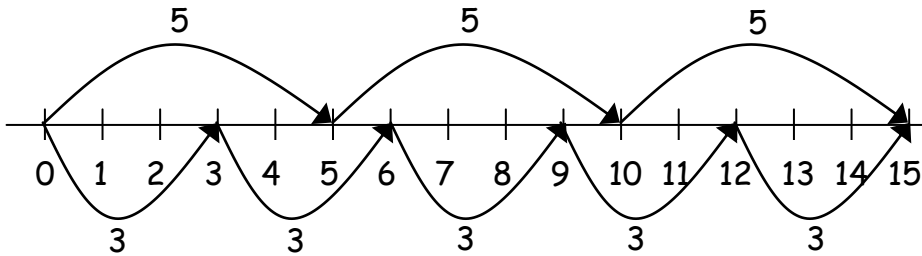
and on a bead bar:

$$5 \times 3 = 5 + 5 + 5$$



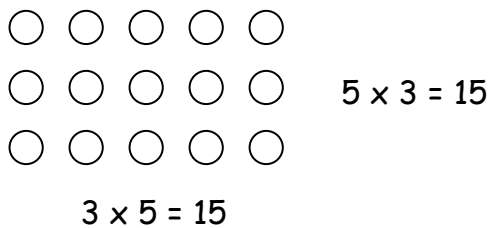
✓ **Commutativity**

Children should know that 3×5 has the same answer as 5×3 . This can also be shown on the number line.



✓ **Arrays**

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.



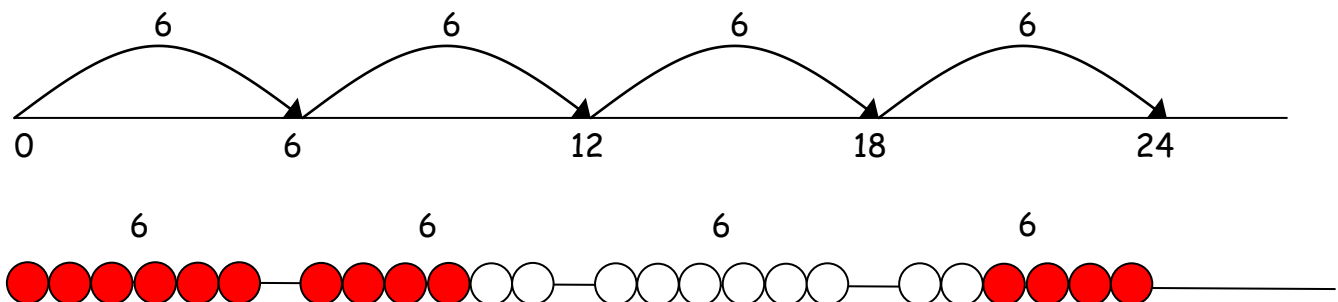
Y3

Children will continue to use:

✓ **Repeated addition**

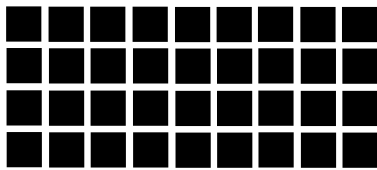
4 times 6 is $6 + 6 + 6 + 6 = 24$ or 4 lots of 6 or 6×4

Children should use number lines or bead bars to support their understanding.



✓ **Arrays**

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.



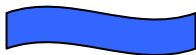
$$9 \times 4 = 36$$

$$9 \times 4 = 36$$

Children will also develop an understanding of

✓ **Scaling**

e.g. Find a ribbon that is 4 times as long as the blue ribbon



5 cm



20 cm

✓ **Using symbols to stand for unknown numbers to complete equations using inverse operations**

$$\square \times 5 = 20$$

$$3 \times \triangle = 18$$

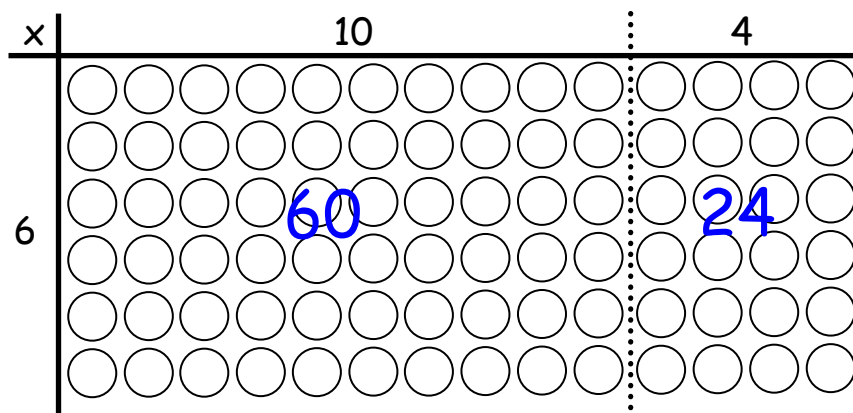
$$\square \times \circ = 32$$

✓ **Partitioning**

$$\begin{aligned} 38 \times 5 &= (30 \times 5) + (8 \times 5) \\ &= 150 + 40 \\ &= 190 \end{aligned}$$

Y4

Children will continue to use arrays where appropriate leading into the grid method of multiplication.



$$(6 \times 10) + (6 \times 4)$$

$$60 + 24$$

$$84$$

Grid method

TO \times O

(Short multiplication - multiplication by a single digit)

$$23 \times 8$$

Children will approximate first

23×8 is approximately $25 \times 8 = 200$

$$\begin{array}{r} \times \quad 20 \quad 3 \\ 8 \quad \boxed{160} \quad \boxed{24} \end{array}$$

$$\begin{array}{r} 160 \\ + \quad 24 \\ \hline 184 \end{array}$$

Y4

Grid method

HTO × O

(Short multiplication - multiplication by a single digit)

$$346 \times 9$$

Children will approximate first

346×9 is approximately $350 \times 10 = 3500$

$$\begin{array}{r} \times \quad 300 \quad 40 \quad 6 \\ 9 \quad \boxed{2700} \quad \boxed{360} \quad \boxed{54} \end{array}$$

$$\begin{array}{r} 2700 \\ + 360 \\ + \quad 54 \\ \hline 3114 \\ \small{11} \end{array}$$

Followed by

$$\begin{array}{r} 27 \\ \times 4 \\ \hline 108 \\ 2 \end{array}$$

$$\begin{array}{r} 273 \\ \times 4 \\ \hline 1092 \\ 21 \end{array}$$

Year 5

TO x TO

(Long multiplication - multiplication by more than a single digit)

$$72 \times 38$$

Children will approximate first

72×38 is approximately $70 \times 40 = 2800$

| | | | |
|----|------|----|-------------|
| x | 70 | 2 | |
| 30 | 2100 | 60 | 2100 |
| 8 | 560 | 16 | + 560 |
| | | | + 60 |
| | | | + <u>16</u> |
| | | | <u>2736</u> |
| | | | 1 |

Long multiplication in expanded form up to 4 digits x 2 digits.

| | | | |
|---|---------------------|---------------|-------------|
| X | 72 | Moving on to: | 72 |
| | <u>38</u> | | <u>38</u> |
| | 16 (8x2) | | 576 |
| | 560 (8x70) | | <u>2160</u> |
| | 60 (30x2) | | <u>2736</u> |
| | <u>2100</u> (30x70) | | 1 |
| | <u>2736</u> | | |
| | 1 | | |

Using similar methods, they will be able to multiply decimals with one decimal place by a single digit number, approximating first. They should know that the decimal points line up under each other.

e.g. 4.9×3

Children will approximate first

4.9×3 is approximately $5 \times 3 = 15$

$$\begin{array}{r} \times \quad 4 \quad 0.9 \\ 3 \quad \boxed{12} \quad \boxed{2.7} \\ \hline \end{array} \qquad \begin{array}{r} 12 \\ + \quad 2.7 \\ \hline 14.7 \end{array}$$

Followed by

Short multiplication up to 4 digits by 1 digit numbers including decimals to 2 decimal places.

$$\begin{array}{r} \times \quad 273.4 \\ \quad \quad 4 \\ \hline 1093.6 \\ 211 \end{array}$$

Using similar methods, they will be able to multiply decimals with up to two decimal places by a single digit number and then two digit numbers, approximating first. They should know that the decimal points line up under each other.

For example:

4.92×3

Children will approximate first

4.92×3 is approximately $5 \times 3 = 15$

$$\begin{array}{r} \times \quad 4 \quad 0.9 \quad 0.02 \\ 3 \quad \boxed{12} \quad \boxed{2.7} \quad \boxed{0.06} \\ \hline \end{array} \qquad \begin{array}{r} 12 \\ + \quad 0.7 \\ + \quad 0.06 \\ \hline 12.76 \end{array}$$

ThHTO × O

(Short multiplication - multiplication by a single digit)

$$4346 \times 8$$

Children will approximate first

4346×8 is approximately $4346 \times 10 = 43460$

| | | | | |
|---|-------|------|-----|----|
| x | 4000 | 300 | 40 | 6 |
| 8 | 32000 | 2400 | 320 | 48 |

$$\begin{array}{r} 32000 \\ + 2400 \\ + 320 \\ + 48 \\ \hline 34768 \end{array}$$

Followed by

Formal short multiplication method

$$\begin{array}{r} 4346 \\ \times \quad 8 \\ \hline 34768 \\ 234 \end{array}$$

Year 6

HTO x TO

(Long multiplication - multiplication by more than a single digit)

$$372 \times 24$$

Children will approximate first

372×24 is approximately $400 \times 25 = 10000$

| | | | | |
|----|------|------|----|--------|
| x | 300 | 70 | 2 | |
| 20 | 6000 | 1400 | 40 | 6000 |
| 4 | 1200 | 280 | 8 | + 1400 |

| | | | | |
|--|--|--|--|--------|
| | | | | + 1200 |
| | | | | + 280 |
| | | | | + 40 |
| | | | | + 8 |
| | | | | <hr/> |
| | | | | 8928 |
| | | | | <hr/> |
| | | | | 1 |

Children will move to formal long multiplication:

$$\begin{array}{r} 24 \\ \times 16 \\ \hline 240 \text{ (10} \times 24\text{)} \\ 144 \text{ (6} \times 24\text{)} \\ \hline 384 \end{array}$$

Ones first:

$$\begin{array}{r} 124 \\ \times 26 \\ \hline 744 \\ 2480 \\ \hline 3224 \end{array}$$

1 1

By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

- they are not ready.
- they are not confident.

Children should be encouraged to approximate their answers before calculating.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

PROGRESSION THROUGH CALCULATIONS FOR DIVISION

MENTAL CALCULATIONS

(ongoing)

These are a **selection** of mental calculation strategies from The National Numeracy Strategy framework:

Doubling and halving

Knowing that halving is dividing by 2

Deriving and recalling division facts

Tables should be taught everyday from Y2 onwards, either as part of the mental oral starter or other times as appropriate within the day.

Year 2 2 times table
 3 times table
 4 times table
 5 times table
 10 times table

Year 3 2 times table
 3 times table
 4 times table
 5 times table
 6 times table
 8 times table
 9 times table
 10 times table

Years 4, 5 & 6 Derive and recall all division facts up to 12×12

Using and applying division facts

Children should be able to utilise their tables knowledge to derive other facts.

e.g. If I know $3 \times 7 = 21$, what else do I know?

$30 \times 7 = 210$, $300 \times 7 = 2100$, $3000 \times 7 = 21\ 000$, $0.3 \times 7 = 2.1$ etc

Dividing by 10 or 100

Knowing that the effect of dividing by 10 is a shift in the digits one place to the right.

Knowing that the effect of dividing by 100 is a shift in the digits two places to the right.

Use of factors

$$378 \div 21 \quad 378 \div 3 = 126 \quad 378 \div 21 = 18$$

$$126 \div 7 = 18$$

Use related facts

Given that $1.4 \times 1.1 = 1.54$

What is $1.54 \div 1.4$, or $1.54 \div 1.1$?

MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.

THE FOLLOWING ARE STANDARDS THAT WE EXPECT THE MAJORITY OF CHILDREN TO ACHIEVE.

YR and Y1

Children will understand equal groups and share items out in play and problem solving. They will count in 2s and 10s and later in 5s.

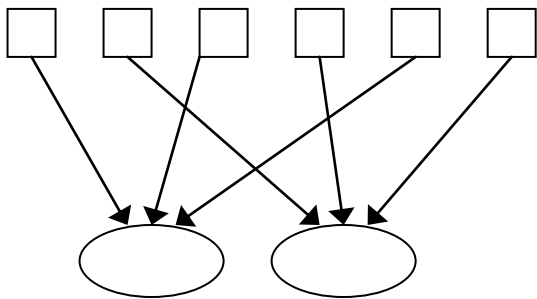


Y2

Children will develop their understanding of division and use jottings to support calculation

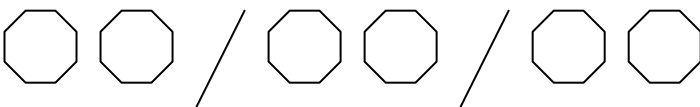
✓ **Sharing equally**

6 sweets shared between 2 people, how many do they each get?



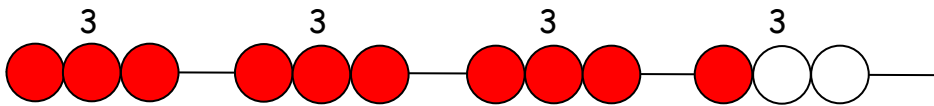
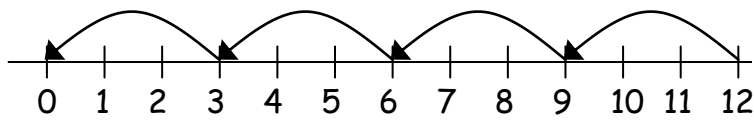
✓ **Grouping or repeated subtraction**

There are 6 sweets, how many people can have 2 sweets each?



- ✓ Repeated subtraction using a number line or bead bar

$$12 \div 3 = 4$$



The bead bar will help children with interpreting division calculations such as $10 \div 5$ as 'how many 5s make 10?'

- ✓ Using symbols to stand for unknown numbers to complete equations using inverse operations

$$\square \div 2 = 4 \qquad 20 \div \triangle = 4 \qquad \square \div \triangle = 4$$

Y3

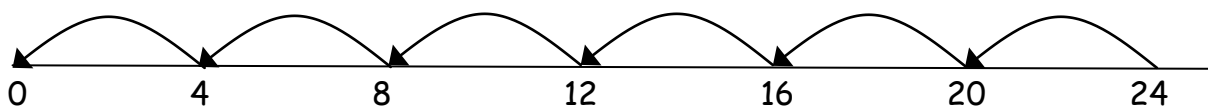
Ensure that the emphasis in Y3 is on grouping rather than sharing.

Children will continue to use:

- ✓ Repeated subtraction using a number line

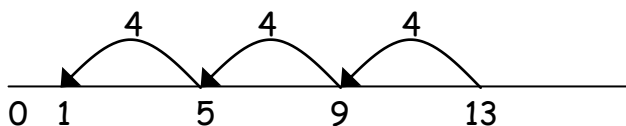
Children will use an empty number line to support their calculation.

$$24 \div 4 = 6$$



Children should also move onto calculations involving remainders.

$$13 \div 4 = 3 \text{ r } 1$$



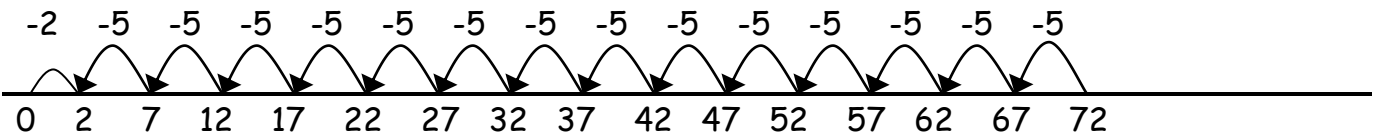
✓ Using symbols to stand for unknown numbers to complete equations using inverse operations

$26 \div 2 = \square$ $24 \div \triangle = 12$ $\square \div 10 = 8$

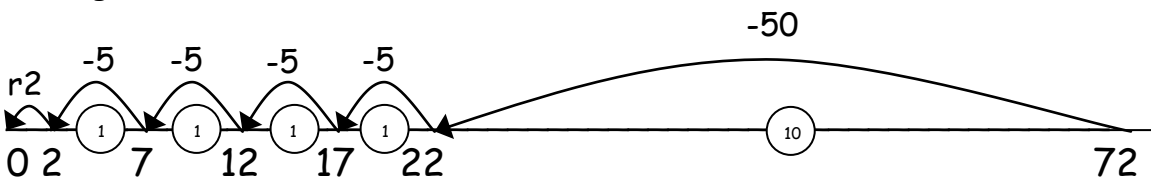
Y4

Children will develop their use of repeated subtraction to be able to subtract multiples of the divisor. Initially, these should be multiples of 10s, 5s, 2s and 1s - numbers with which the children are more familiar.

$72 \div 5 = 14r2$



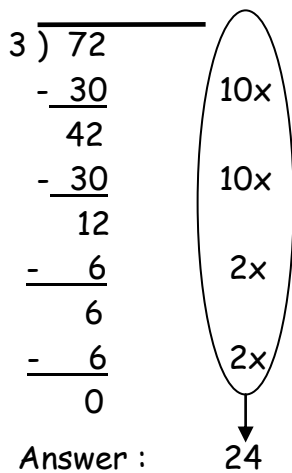
Moving onto:



Then onto the vertical method:

Short division $70 \div 3$

$72 \div 3 = 24$



Leading to subtraction of other multiples.

$$96 \div 6 = 16$$

$$\begin{array}{r} 16 \\ 6 \overline{) 96} \\ \underline{- 60} \\ 36 \\ \underline{- 36} \\ 0 \end{array}$$

10x
6x

Answer : 16

Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.

Children need to be able to decide what to do after division and round up or down accordingly. They should make sensible decisions about rounding up or down after division. For example $62 \div 8$ is 7 remainder 6, but whether the answer should be rounded up to 8 or rounded down to 7 depends on the context.

e.g. I have 62p. Sweets are 8p each. How many can I buy?

Answer: 7 (the remaining 6p is not enough to buy another sweet)

Apples are packed into boxes of 8. There are 62 apples. How many boxes are needed?

Answer: 8 (the remaining 6 apples still need to be placed into a box)

Y5

Children will continue to use written methods to solve short division for numbers up to 4 digits by a one digit number.

Children can start to subtract larger multiples of the divisor, e.g. 30x

Short division HTO \div O

$$196 \div 6 = 32r4$$

$$\begin{array}{r} 32 \text{ r } 4 \\ 6 \overline{) 196} \\ \underline{- 180} \\ 16 \\ \underline{- 12} \\ 4 \end{array}$$

30x
2x

Answer : 32 remainder 4 or 32 r 4

Moving into formal methods (short formal division method)

$$\begin{array}{r} \underline{32r4} \\ 6 \overline{)1916} \end{array}$$

Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.

Children need to be able to decide what to do after division and round up or down accordingly. They should make sensible decisions about rounding up or down after division. For example $240 \div 52$ is 4 remainder 32, but whether the answer should be rounded up to 5 or rounded down to 4 depends on the context.

Y6


Children will continue to use written methods to solve short division $TO \div O$ and $HTO \div O$ and then into long division with numbers up to 4 digits divided by a 2 digit whole number.

Long division $HTO \div TO$

$$972 \div 36 = 27$$

$$\begin{array}{r} 27 \\ 36 \overline{) 972} \\ \underline{- 720} \\ 252 \\ \underline{- 252} \\ 0 \end{array}$$

Answer : 27



Any remainders should be shown as fractions, i.e. if the children were dividing 32 by 10, the answer should be shown as $3 \frac{2}{10}$ which could then be written as $3 \frac{1}{5}$ in its lowest terms.

Extend to decimals with up to two decimal places. Children should know that decimal points line up under each other.

$$87.5 \div 7 = 12.5$$

| | |
|-----------------------|------------------------------------|
| 12.5 | |
| 7 $\overline{) 87.5}$ | |
| - 70.0 | 10x

2x

0.5x |
| 17.5 | |
| - 14.0 | |
| 3.5 | |
| - 3.5 | |
| 0 | |
| Answer : | ↓
12.5
+ - + - + - + - + - + |

By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

- they are not ready.
- they are not confident.

Children should be encouraged to approximate their answers before calculating. Children should be encouraged to check their answers after calculation using an appropriate strategy.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.